

## SOLID AND LIQUID EQUATION OF STATE FOR INITIALLY POROUS ALUMINUM WHERE SPECIFIC HEAT AND $\gamma/v$ ARE CONSTANT

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**Abstract.** A porous solid's initial state is off the thermodynamic surface of the solid to start with but when pressure is high enough to cause total pore collapse or crush up, the final states are on the thermodynamic surfaces. The Hugoniot for the fully compacted initially porous solid has pressure above the principal Hugoniot at the same  $v$  due to an additional thermal pressure contribution. There are a number of ways to define this hotter Hugoniot, which can be referenced to other thermodynamic paths on this thermodynamic surface. The choice here was to use the Rose-Vinet isotherm to define a consistent thermodynamic surface for the solid and melt phase of 6061 aluminum where specific heat is constant for the P-v-T space of interest. Analytical equations are developed for principal Hugoniots  $P_H(v)$  and  $T_H(v)$ .

**Keywords:** Equation of state, porous metals, 6061 aluminum, melting

**PACS:** 62.50.Ef, 64.30.Ef, 64.70.kd

### INTRODUCTION

This consistent equation of state (EOS) development is similar to that of Cowperthwaite [1] and Johnson, Asay, and Hayes [2] two-phase EOS for bismuth where the second phase is the melted phase. In addition, Asay and Hayes [3] conducted experiments on 41% porous aluminum and developed a thermodynamically consistent EOS surface with  $C_v(T)$  for calculating shocked porous aluminum thermodynamic paths. Thermodynamic consistency is important for EOS use in codes where derivatives of EOS are used.

### EOS DEVELOPMENT

A useful thermodynamic identity is

$$\left(\frac{\partial P}{\partial T}\right)_v = C_v \frac{\gamma}{v} \quad (1)$$

which can be integrated if  $C_v \gamma/v$  is constant giving

$$P(x, T) = C_v \frac{\gamma_o}{v_o} (T - T_o) + P_{T_o}(v) \quad (2)$$

This general expression for P,v,T on the thermodynamic surface requires an isotherm  $P_{T_o}$  for the reference path. A recent review [4] of high-pressure isothermal equations of state shows that the third order Rose-Vinet isotherm is quite good up to 90% compression, which is below the point where a material becomes an electron gas. Extreme compression states are not considered here. The Rose-Vinet isotherm [4] is

$$P_{T_o}(x) = 3B_{T_o}(1-x)x^{-2} \exp\left[\frac{3}{2}(B_{T_o}-1)(1-x)\right] \quad (3)$$

where  $B_{T_o}$  is constant bulk modulus at STP,  $B'_{T_o}$  is the pressure derivative of the isothermal bulk modulus but is a fitted constant as used here, and  $x$  is  $(v/v_o)^{1/3}$ .

The general expression [5] for E(v,P) is

$$E(v,P) = E_o(v_o,0) + \frac{v_o}{\gamma_o} (P - P_{T_o}) - T_o C_v \gamma (1 - x^3) - \frac{9v_o B_{T_o}}{B_1} \left[ \left( x - 1 + \frac{1}{B_1} \right) \exp(B_1(1-x)) - \frac{1}{B_1} \right] \quad (4)$$

where  $B_1 = 3/2(B'_{T_o})$ . The  $P_H(v)$  expression [5] for the fully compacted formerly porous solid is

$$P_H(v) = \frac{-P_{T_o}(v) - T_o C_v \gamma_o (1 - x^3) + \frac{9v_o B_{T_o}}{B_1} \left[ \left( 1 - x - \frac{1}{B_1} \right) \exp(B_1(1-x)) + \frac{1}{B_1} \right]}{\frac{(v_{oo} - v_o x^3)}{2} - \frac{v_o}{\gamma_o}} \quad (5)$$

Equation 5 was obtained by setting  $E - E_o$  for a totally shock compacted porous material with initial volume  $v_{oo}$  equal to Eq. 4, which results in Eq. 5 for the hotter Hugoniot than the principal Hugoniot for the same v.

Paths on the thermodynamic surface are obtained by using  $E(v,P) - E_o(v_o, P=0)$  for the path to eliminate  $E - E_o$  from Eq. 4. The surface energy of the pores in a porous material is small compared to shock internal energies, so can be ignored. The Rankine-Hugoniot equation for this porous solid with initial state ( $P=0, v_{oo}, T_o$ ) is  $E_H - E_o = P_H(v_{oo} - v)/2$ . Note that Eq. 5 also gives the principal solid Hugoniot [5] with initial state ( $P=0, v_o, T_o$ ) if  $v_o$  is substituted for  $v_{oo}$ .

Solving Eq. 2 for T and using Eq. 5 for  $P = P_H(v)$  for the hotter Hugoniot results in its  $T_H$

$$T_H(v, P_H) = T_o + \left[ P_H(v) - P_{T_o}(v) \right] \frac{v_o}{C_v \gamma_o} \quad (6)$$

If melting occurs at an infinite rate and the shock in aluminum is strong enough to have an end state in the liquid phase, an equation for  $P_H$  can be developed similar to Eq. (5) with initial state  $P=0, v_{oo}$ . For the melted phase  $E(v, P) - E(v_o^m(T_o^m), P=0) = E(v, P) - E_o^s + T_o^m \Delta S + C_p^s(T_o^m - T_o^s)$ . Along the

isobar  $P=0, v_o^m = 0.4211 \text{ cm}^3/\text{g}$ , and  $T_o^m = 933 \text{ K}$ . This  $P_H$  equation for the melted aluminum can be obtained by subtracting  $T_o^m \Delta S + C_p^s(T_o^m - T_o^s)$  from the numerator of Eq. 5 and changing  $v_o$  to  $v_o^m$  and  $\gamma_o$  to  $\gamma_o^m$  in Equation 5.

The equations of state parameters for 6061 aluminum solid and melted/liquid phase are given in Table 1.

**Table 1.** Equation of state parameters for solid and liquid 6061 aluminum

| Parameter                               | Solid                            | Liquid | References |
|---|----------------------------------|--------|------------|
| $S_o$ (J/kg K)                          | 0                                | 521    | 3          |
| $T_o$ (K)                               | 298                              | 933    | 6          |
| $v_o$ (cm <sup>3</sup> /g)              | 0.3690                           | 0.4211 | 6          |
| $\gamma_o$                              | 2.02                             | 0.814  | 5          |
| $B_{oT}$ (Mb)                           | 0.730                            | 0.521  | 7,8        |
| $B'_{T_o} = dB_{oT}/dP$                 | 4.54                             | 4.54   | 7          |
| Volume $\alpha \times 10^{-5}/\text{K}$ | 6.93                             | 4.23   | 8          |
| $C_v$ (J/g-K)                           | 0.925<br>(Dulong-Petit limit 3R) | 1.14   | 9,10       |

Aluminum's P-T melt line has been measured using various techniques. The entire set of

available melting data for aluminum up to 1.25 Mb was collected [2, 10-14], including some recent measurements, that when fitted to the Simon melting law results in

$$T_M = 933 K \left( \frac{P}{0.06941 \text{ Mb}} + 1 \right)^{0.5593} \quad (7)$$

The aluminum P-T melt line needs to be put into P-v space for this equation of state surface to be used in the SHAMRC code. Using the aluminum melt line of Eq. 7 with Eq. 2 defined for the separate phases gives these P-v curves.

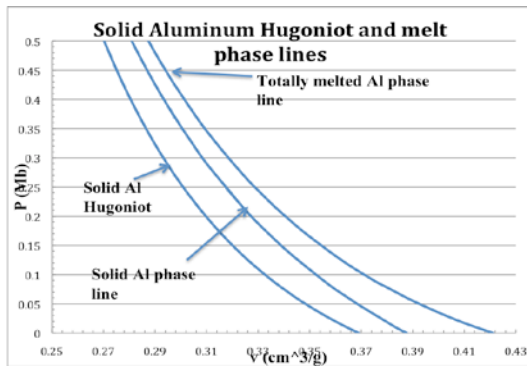
The P-v phase line numerical results for the solid boundary and the totally melted boundary are fitted to third order polynomials for use in the SHAMRC code. The aluminum melt P-v phase line for the solid boundary in Mb is

$$P = -157.33 v^3 + 186.28 v^2 - 76.042 v + 10.649 \quad (8)$$

The phase line for the totally melted aluminum in Mb is

$$P = -103.7 v^3 + 132.43 v^2 - 58.068 v + 8.7113 \quad (9)$$

Figure 1 gives the Hugoniot for solid aluminum along with the melt phase lines. It is clear that the solid material will not melt under shock compression for stresses near 0.5 Mb. So to melt aluminum under shock compression for stresses of 0.5 Mb or less requires that the aluminum be porous.



**Figure 1.** Aluminum solid Hugoniot and phase boundaries for solid and liquid phase

This choice of the Rose-Vinet isotherm as the thermodynamic surface reference path is a departure from the approach used by Asay and Hayes [3] who used the Murnaghan isotherm as the reference path, which is only accurate over a small compression range. They also let specific heat vary as a function of volume and temperature, which gives a more accurate  $C_v$  near room temperature.

Asay and Hayes [3] used the release wave velocity dropping from longitudinal/elastic velocity to bulk sound velocity as the criterion for where melting occurred. They reported that the hot compacted Hugoniot for 41% porous 1100 aluminum samples reached the melt line at around 0.075 Mb and 1340 K. This is in good agreement with our calculated hot Hugoniot intersecting the newly fitted phase line at 0.0777 Mb and  $T_M = 1419$  K. Most of the difference between our results and Asay and Hayes is due to our use of an updated melt line.

Another very important conclusion of the Asay and Hayes study was that the Hugoniot in the mixed phase region followed the metastable solid Hugoniot curve extension, indicating that melting was slower than the rise-time of the shock front. These metastable states are important for doing wave profile calculations. While the kinetics of melting is not accurately known, melting appears to occur in the microsecond time frame upon the release of pressure.

## CONCLUSIONS

Under the assumption of constant  $C_v$  and  $\gamma/v$ , thermodynamically consistent analytical equations are derived for  $P_H(v)$  and  $T_H(v)$ . The Rose-Vinet isotherm good up to about 90% compression is used as the reference path to define the thermodynamic surface. Porous metals fully compacted by shock waves will be at temperatures above room temperature, which is temperature space where the constant Dulong-Petit [9] value for  $C_v$  is asymptotically reaching this value. The commonly used approximation of  $\gamma/v$  being a constant was used to obtain the analytical equations for the thermodynamic paths. To improve the accuracy of this EOS future work needs to include

the volume dependence of  $\gamma$  from measurements or calculations.

Having  $E(v)$ ,  $P_H(v)$ , and  $P_S(v)$  as analytical equations makes them easier to use in Hydro-codes and in numerical solvers.

The Hugoniot temperature and pressure for 41% porous aluminum gives comparable results as, Asay and Hayes work [3] at low pressures. However, the equations given in this paper will be more accurate at higher pressures than this earlier work due to the use of the measured isotherm fitted to the Rose-Vinet equation [7]. An additional contribution of this paper is the improved melt line of aluminum that fit more acceptable data than previous melt lines have used.

#### ACKNOWLEDGEMENTS

The authors of this report would like to acknowledge many useful discussions with Hank Happ at ARA on EOS format needed for SHAMRC code. Dr. Dennis Grady (ARA) provided insight into the history of the Rose-Vinet equation of state, Dr. Joseph Zaug (LLNL) brought reference 7 on accurate high-pressure isotherms to the author's attention. Dr. Kibong Kim (AER) is acknowledged for getting J. Forbes interested in this task. This work was funded from the Office of Naval Research (ONR) through the High Energy Conventional Energetics program (HECE) contract number N00014-09-C-0080 (Dr. Clifford Bedford).

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